Regimes of Fluidization for Large Particles

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Most applications of fluidized beds involve gas fluidization of fine and moderately fine particles (less than 500 μm in diameter). Gas flow, heat transfer, and the movement of solids have been studied for beds of fine particles, and reasonably good predictive correlations have been suggested. Models based on the behavior of bubbles in beds of fine solids have been developed and used successfully to describe the performance of a fluidized bed as a chemical reactor.

Fluidized beds with coarse particles have become more important in recent years with the increased interest in fluidized bed coal combustion. Fluidized bed combustors typically use large limestone particles (greater than 1 000 μm in diameter) to scavenge the sulfur dioxide produced by burning sulfur-containing coal. The air percolates through the emulsion phase in such beds of coarse particles with a velocity an order of magnitude greater than it has in fine particle beds. Because of this, the nature of fluidization is changed; gas may rise through the interstices in the emulsion at a velocity greater than the rise velocity of the bubbles. Also, bubbles grow more quickly as they rise (an effect that increases with increasing gas velocity), so that slugging can occur in relatively wide and shallow beds. The first condition is usually referred to (somewhat misleadingly) as the slow bubble regime (Kunii and Levenspiel, 1969), not because the bubbles move slower than they would in a fine particle bed, but because the emulsion gas moves faster than the bubbles. The tendency to form large bubbles has been called apparent slugging (Canada et al., 1976), particularly when the bubbles form in a bed which is too shallow to allow a train of true slugs to develop. We will refer to this as the rapidly growing bubble regime. At higher velocities still another flow regime has been found in which large gas voids or bubbles are absent. It has been named the turbulent regime (Yerushalmi et al., 1976*a*), and it occurs both in large and small particle beds (Yerushalmi et al., 1976*b*; Canada et al., 1976). There is not much published data on the limiting velocity for the turbulent regime, but it does seem to occur at a smaller multiple of the minimum fluidizing velocity (u/u_{mf}) for coarse particle beds.

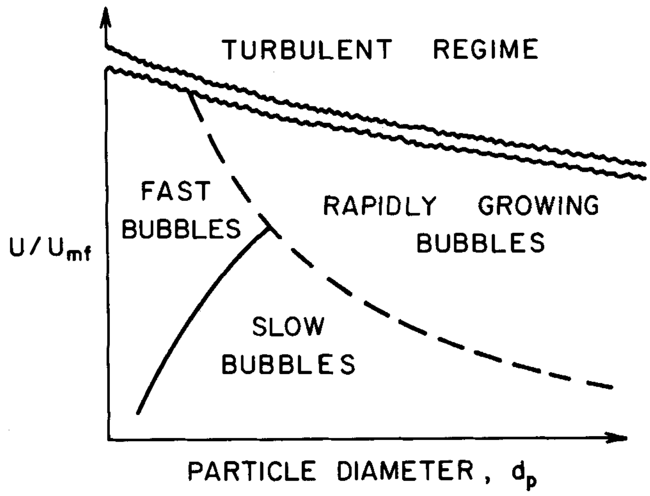

Fig. 1. Map of large particle fluidization regimes.

Since rather limited data are available at present, the purpose of this note is to chart and sketch important trends and observations useful to current and future research and application. An attempt is made to describe the above regimes and define criteria for distinguishing between them in fluidized beds without internals.

DESCRIPTION OF REGIMES

Figure 1 represents a qualitative map of the regimes encountered when large particles are fluidized.

A *fast bubble* (Figure 2*a*) exists in a bed in which the gas percolating through the emulsion is moving upward slower than the bubble:

$$u_f < u_{br} \tag{1}$$

Gas enters the lower part of the bubble and leaves at the top. However, it is then swept around and returns forming a captive cloud around the bubble. Fast bubbles are characteristic for beds of fine particles. The various bubble and emulsion models (Rowe, 1964; Partridge and Rowe, 1966; Latham et al., 1968; Kunii and Levenspiel, 1969; Kato and Wen, 1969; Fryer and Potter, 1972) were developed for this regime. All these models assume that most of the gas which passes through a bubble is recirculated back to it, and they define mass transfer coefficients which are based on a limited exchange of gas between locally well-mixed bubble and emulsion regions.

A *slow bubble* (Figure 2*b*) is encountered in a bed in which the interstitial velocity of the gas exceeds the rising velocity of the bubble:

$$u_f > u_{br} \tag{2}$$

Here the gas uses the bubble as a convenient shortcut on its way through the bed. Slow bubbles are expected to appear when large particles are fluidized, as the interstitial velocity has to be high in order to achieve the fluidized state. This regime has recently been investigated by McGrath and Streatfield (1971), Cranfield and Geldart (1974), Cranfield (1976), Canada et al. (1976), and McGaw (1977).

Because of the relative absence of recirculating gas, the bubble-emulsion models are not adequate for slow bubbles. There is evidence that solids movement is different

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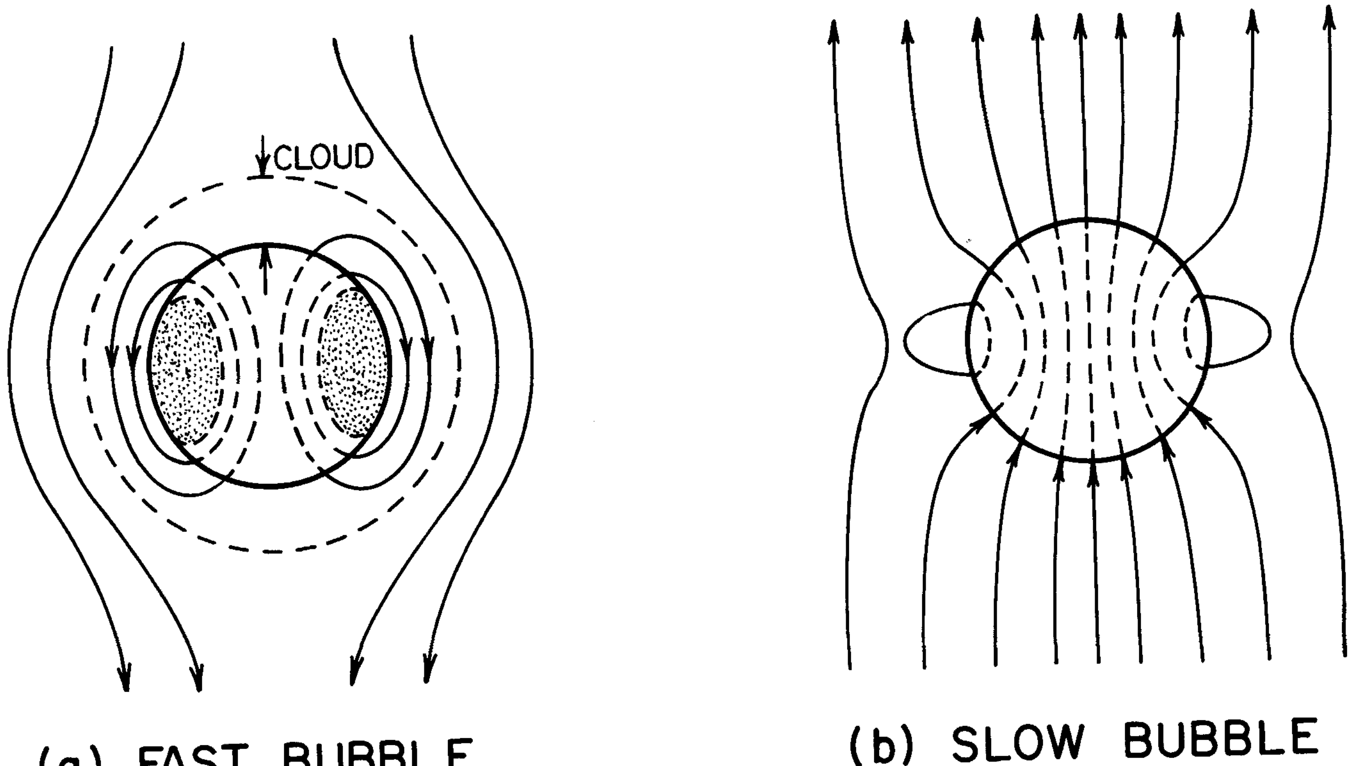

Fig. 2. Differences in gas streamlines between a fast bubble and a slow bubble.

too; slow bubbles have significantly smaller wakes of solids trailing behind them than do fast bubbles, and the extent of solids mixing and gas backmixing is therefore far lower than in small particle fluidized beds (Cranfield and Geldart, 1974; Cranfield, 1976; McGaw, 1977). Heat transfer correlations for fine particles, which are based on the assumption that heat is transferred to surfaces primarily by packets of solids, are not likely to apply either. Because of high gas velocities in large particle beds, heat transfer through the gas film between the surface and the particle (the gas convective component) will be a much more dominant factor than heat transfer due to the motion of particles.

The *rapidly growing bubble* regime is reached at higher superficial gas velocities when the bubble growth rate is of the same magnitude as the bubble rise velocity; that is

$$\frac{d(d_b)}{dt} \cong u_b \tag{3}$$

This regime is characterized by large pressure drop oscillations, with amplitudes comparable to a significant portion of the mean pressure drop through the bed. Bubbles in this regime have been observed in shallow beds of large particles ($H < D$) by Cranfield and Geldart (1974), Canada et al. (1976), and McGaw (1977). The diameters of erupting bubbles can reach one-half to three-fourths of the bed height. In deeper beds, the bed-diameter bubbles form, and a transition to slugging takes place (Geldart et al., 1976).

The bubble and emulsion models do not account for an accumulation of mass in the bubble; it is assumed negligible compared to gas throughflow. For the rapidly growing bubble regime, accumulation in a bubble is comparable in magnitude to throughflow. Thus, it follows that the bubble and emulsion models should not be used for this regime.

The *turbulent regime* occurs at relatively high velocities. Surprisingly, pressure drop fluctuations decrease significantly. Models for representing turbulent beds have yet to be developed.

CRITERIA FOR DISTINGUISHING BETWEEN REGIMES

In this analysis the interstitial gas velocity is calculated from

$$u_f = \frac{u_{mf}}{\epsilon_{mf}} \cong u_e, \quad \text{with} \quad \epsilon_{mf} \cong 0.4 \tag{4}$$

which ought to be a reasonable approximation for large particle beds. Interstitial velocities measured by McGaw (1977) in beds containing particles from 1.8 to 3.9 mm agree well with Equation (4) up to superficial velocities of about 2 u_{mf}. The minimum fluidizing velocity is given by (Wen and Yu, 1966)

$$u_{mf} = \frac{\mu}{d_p \rho_g} \left[(33.7)^2 + 0.0408 \frac{d_p{}^3 \rho_g (\rho_s - \rho_g) g}{\mu^2} \right] - 33.7 \tag{5}$$

The equation for single bubble rise velocity

$$u_{br} = 0.711 (g d_b)^{1/2} = 22.26\, d_b{}^{1/2} \tag{6}$$

was proposed by Davidson and Harrison (1963). Single bubbles are not often encountered in commercial fluidized beds; rather, swarms of bubbles move continuously through the bed. The rise velocity of bubbles in a swarm is given by the equation of the same authors:

$$u_b = u - u_{mf} + u_{br} \tag{7}$$

However, bubble velocity relative to the surrounding emulsion phase remains u_{br}, and u_{br} is therefore used in the slow to fast bubble transition criterion (Bar-Cohen et al., 1977).

The only known bubble growth equation for beds of large particles has been developed by Cranfield and Geldart (1974):

$$d_b = 0.0326 (u - u_{mf})^{1.11}\, h^{0.81} \tag{8}$$

Since the authors observed that bubbles do not originate at the distributor, their equation applies for $h > 5$ cm. The original equation contained the minimum bubbling velocity u_{mb} instead of the minimum fluidizing velocity u_{mf}, but in fluidized beds of coarse particles, there is very little difference between the two (Broadhurst and Becker, 1975; Yerushalmi et al., 1976*b;* McGaw, 1977). This equation was developed on the basis of data in shallow beds but was also used and verified in deep beds by Geldart et al. (1976). It differs significantly from bubble growth equations for beds of fine particles (Rowe, 1976), which is an indication of the difference in fluid dynamics between bubbles in large particle and small particle beds.

Equations (4), (5), (6), and (8) can be combined with Equations (1) and (2) to obtain a criterion for determining whether bubbles are fast or slow. This is represented graphically by solid lines in Figures 3 and 4 for fluidized beds of limestone ($\rho_s = 2.93$ g/cm^3) at room temperature and atmospheric pressure, with bed height as a parameter. For a given bed height the region under the corresponding solid line designates the slow bubble region, while the points above the line lie in the fast bubble region. With decreasing density of the fluidized material, the border line between slow and fast bubbles moves downward on both graphs.

At this point the question arises as to whether it is possible, by increasing the superficial velocity, to reach the fast bubble regime for any particle size. In deeper beds ($H/D > 1$), one must take into account the transition to slugging which occurs at higher excess gas velocities. However, for coal combustion and many other coarse particle bed applications, shallow beds are of greater significance owing to some fundamental design constraints (Anson, 1976; Cranfield, 1976).

Shallow Beds

Beds with $H < D$ cannot exhibit slugging in the classical sense even at very high superficial velocities, regardless of whether small or large particles are fluidized (Zenz and Othmer, 1960; Baeyens and Geldart, 1974; Geldart et al., 1976). This could lead to the conclusion that by increasing the gas throughput the fast bubble regime could be achieved. However, for beds of large particles this is not the case.

As the excess gas velocity is increased, the bubbles will grow more rapidly. At a certain value of $u - u_{mf}$, the bubble diameter will grow as quickly as the bubble rises, and Equation (3) will be satisfied. This condition can be represented as

$$\frac{d(d_b)}{dh}\,\frac{dh}{dt} = u_b \qquad (9a)$$

which is equivalent to

$$\frac{d(d_b)}{dh} = 1 \qquad (9b)$$

Using Equation (8) for bubble growth, the criterion for rapidly growing bubbles takes the form of

$$u - u_{mf} = 21.85\ h^{0.17} \qquad (10)$$

The boundary of the rapidly growing bubble regime for different bed heights is represented by broken lines in Figures 3 and 4. These lines can be considered valid for $d_p > 0.5$ mm, since Equation (8) applies for coarse particles only. The boundary corresponds to what has been described as the onset of violent oscillations (Cranfield and Geldart, 1974) or the transition from gentle bubbling to an apparent slugging behavior (Canada et al., 1976). The point of intersection of this boundary with the slow bubble-fast bubble boundary for a given bed height (point A) determines the limiting particle diameter d_{pL} beyond which it is impossible to obtain fast bubbles in the bed without undergoing a transition to the regime of rapidly growing bubbles.

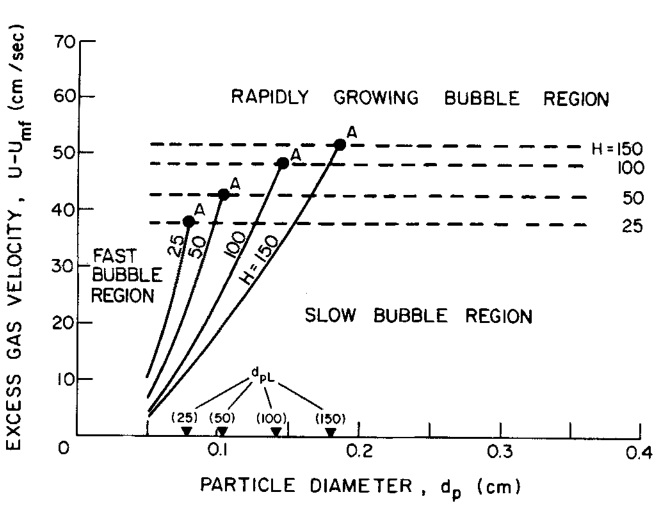

Fig. 3. A graphical criterion for determining bubble regimes in large particle beds of limestone ($\rho_s = 2.93$ g/cm^3) at room temperature and atmospheric pressure, in terms of the excess gas velocity.

The broken lines in Figure 3 agree very well with data of Canada et al. (1976) who fluidized large glass beads (density identical to that of limestone) in beds with diameters of 30 and 60 cm and several different heights. (Their experiments were conducted at room temperature and atmospheric pressure.) For example, for a bed height of 25 cm, Figure 3 shows that a transition to the rapidly growing bubble regime occurs at $u - u_{mf} = 37$ cm/s, while experimental data indicate a value of 35 to 40 cm/s. With a bed height of 50 cm, the respective values are 43 cm/s (Figure 3) and 42 to 45 cm/s (data).

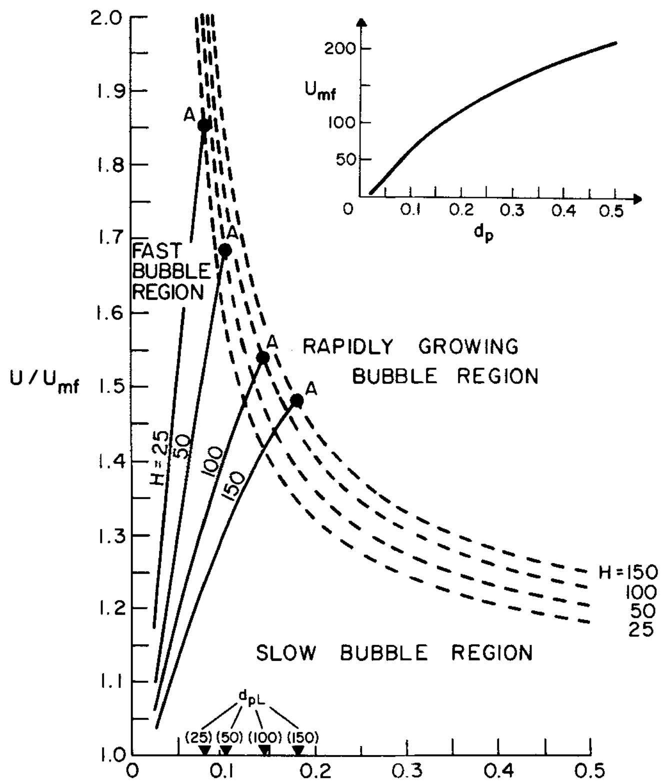

Fig. 4. A graphical criterion for determining bubble regimes in large particle beds of limestone ($\rho_s = 2.93$ g/cm^3) at room temperature and atmospheric pressure, in terms of the ratio u/u_{mf}.

Figure 4 indicates that in beds of coarse particles the rapidly growing bubble regime is reached at relatively low u/u_{mf} ratios. The limiting ratio for this regime decreases with increasing particle size. Figures 3 and 4 show that the fast bubble region is limited to a rather narrow range of particle sizes and superficial velocities.

With a significant increase in the superficial gas velocity, the rapidly growing bubble regime can undergo a transition to the turbulent regime. Canada et al. reported transition values as fractions of u_t, which turn out to be 8 to 9 u_{mf} for 0.65 mm particles and 2.5 u_{mf} for 2.6 mm particles.

Deep Beds

When bed height is such that it enables bed diameter bubbles to form at higher superficial velocities (that is, when $H > D$), a transition to real slugging takes place. If the excess gas velocity exceeds the value obtained from Equation (10), slugs will certainly appear in the upper portions of the bed. However, slugs will appear even at lower values of $u - u_{mf}$ (lower bubble growth rate) provided the bed is sufficiently deep. Therefore, the broken lines in Figures 3 and 4 represent an upper limit for determining d_{pL}. As bed depth increases, the limiting particle diameter decreases and indicates a transition to slugging. The height in the bed where slugs appear at a particular excess gas velocity can be determined from Equation (8) by setting $d_b = kD$, with $k \leq 1$ depending on whether wall slugs or solids slugs (Geldart et al., 1976) come into play.

Other Effects

Fluidized bed coal combustion will likely take place at a temperature of approximately 1 500°F. There are as yet no available data on bubble growth in large particle beds at high temperatures. Geldart and Kapoor (1976) and Yoshida et al. (1976) have reported that bubble size in small and intermediate particle beds tends to decrease somewhat as temperature is increased. If bubble growth in beds of large particles at high temperatures is indeed slower than at room temperature, the limiting particle diameter would move toward slightly higher values.

Atmospheric fluid bed combustors or boilers are likely to contain an immersed array of heat exchange tubes; thus, it will be necessary to include the effect of tube spacing, orientation, and density on fluidization characteristics.

CONCLUSION

Caution must be exercised when fluidized beds of large particles are modeled. The misuse of existing bubble and emulsion models can lead to erroneous results because beds of coarse particles are likely to be in the slow bubble or the rapidly growing bubble regime, possibly slugging, and for sufficiently high velocities in the turbulent regime. For all of the regimes except the fast bubble regime, new models need to be developed.

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NOTATION

D = bed diameter, cm
d_b = bubble diameter, cm
d_p = particle diameter or equivalent diameter, cm
d_{pL} = limiting particle diameter; that is, diameter above which it is impossible to have fast bubbles in the bed without a transition to rapidly growing bubbles (apparent slugging) or slugging, cm
g = gravitational constant, cm/s^2
H = bed height, cm
h = level in bed measured from the distributor, cm
u = superficial gas velocity, cm/s
u_b = rise velocity of bubble in a swarm, cm/s
u_{br} = rise velocity of a single bubble, cm/s
u_e = interstitial gas velocity, cm/s
u_f = interstitial gas velocity relative to particles, cm/s
u_{mb} = minimum bubbling velocity, cm/s
u_{mf} = minimum fluidizing velocity, cm/s
u_t = terminal velocity of particle, cm/s
$u - u_{mf}$ = excess gas velocity, cm/s
ϵ_{mf} = voidage at minimum fluidization
μ = gas viscosity, g/cm s
ρ_g = gas density, g/cm^3
ρ_s = particle density, g/cm^3

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The Critical Radius Effect with a Variable Heat Transfer Coefficient

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Consider a solid object surrounded by a fluid. Heat transfer to or from the object requires a conduction-convection balance at the surface of the object. If the object has a curved surface (such as a circular cylinder or a sphere) and is smaller than a certain critical size, then adding insulation to the surface of the object has the effect of increasing rather than decreasing the heat transfer. This phenomenon, commonly called the critical radius effect, has been known for some time (for example, see Kreith, 1958), and the classical results for the critical radius are

$$r_{\text{critical}} = \begin{cases} \dfrac{k}{h} & \text{for a circular cylinder} \\ \dfrac{2k}{h} & \text{for a sphere} \end{cases} \tag{1}$$

Equation (1) requires that both k and h be constants, and whereas k is a physical property and can logically be taken to be constant, we know that h depends on various parameters (including the radius of the object) and in general is not a constant. Sparrow (1970) developed the formulas for the critical radius for cylinders and spheres for the case where h is given by

$$h = H\, r^{-m}\, (T_o - T_\infty)^n \tag{2}$$

where m and n are both ≥ 0, and H is a constant. In this situation, the formula for the critical radii becomes

$$r_{\text{cylinder}} = \frac{\left[\left(\dfrac{1-m}{1+n}\right)\left(\dfrac{k}{H}\right)\right]^{1/(1-m)}}{(T_o - T_\infty)^{n/(1-m)}} \tag{3}$$

$$r_{\text{sphere}} = \frac{\left[\left(\dfrac{1-m/2}{1+n}\right)\left(\dfrac{2k}{H}\right)\right]^{1/(1-m)}}{(T_o - T_\infty)^{n/(1-m)}} \tag{4}$$

Equation (2) is accurate only within the turbulent regime. However, correlations currently exist which span both laminar and turbulent free convection. Churchill and Chu (1975) have developed the following relationship for free convection around a horizontal cylinder:

$$h = \frac{k_f}{2r}\left\{0.60 + 0.387\left[\frac{GrPr}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{9/16}\right]^{16/9}}\right]^{1/6}\right\}^2 \tag{5}$$

for $10^{-5} < GrPr < 10^{12}$, where Pr is the dimensionless Prandtl number of the surrounding fluid, and Gr is the dimensionless Grashof number. Yuge (1960) has developed the following equation for free convection around spheres:

$$h = \frac{k_f}{2r}\,(2 + 0.392\, Gr^{1/4}) \tag{6}$$

for $1 \leq Gr \leq 10^5$.

Equations (5) and (6) can be generalized in a manner similar to Equation (2), resulting in

$$h = \left[\frac{C_1}{\sqrt{r_o}} + C_2(T_o - T_\infty)^{1/6}\right]^2 \tag{7}$$

for a horizontal cylinder and

$$h = \frac{C_3}{r_o} + C_4(T_o - T_\infty)^{1/4}\, r_o^{-1/4} \tag{8}$$

for a sphere. The influence of the laminar regime gives Equations (7) and (8) a considerably different form from Equation (2). Equations (5) and (6) can now be used in the heat balance at the object's surface, and the critical radius can be determined by setting

$$\frac{dq}{dr} = 0 \quad \text{at} \quad r = r_{\text{critical}} \tag{9}$$

and solving for r_{critical}. When this is done, one gets (for details, see Balmer and Strobusch, 1977):

Cylinder

$$r_{\text{critical}} = \left(\alpha + \frac{a}{b}\right)^2 \tag{10}$$

where α is one of the real roots of the cubic equation:

$$\alpha^3 + \frac{a}{b}\alpha^2 + \left(\frac{3a^2 - k}{b^2}\right)\alpha + \left(\frac{a}{b}\right)^3 = 0 \tag{11}$$

and

$$a = 0.4243\,(k_f)^{1/2} \tag{12}$$

$$b = 0.2737\,(k_f)^{1/2}\left[\frac{8g\beta(T_o - T_\infty)Pr/(\nu^2)}{\left[1+\left(\dfrac{0.559}{Pr}\right)^{9/16}\right]^{16/9}}\right]^{1/6} \tag{13}$$

Sphere

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